13. $\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n}\right)$ diverges by n^{th} term test

$\infty \cdot 0$

n^{th} term test : $\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = 1$

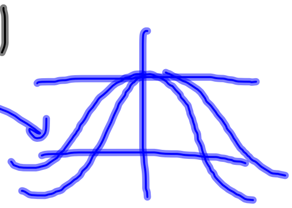
$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \quad \frac{0}{0} \quad = 1$$

let $\theta = \frac{1}{n}$

as $n \to \infty$

$\theta \to 0$

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$



seq. converges to 1

series diverges

Feb 22-1:08 PM

converges by comparison direct comparison test

16. $$\sum_{n=1}^{\infty} \frac{5n^3-3n}{n^2(n+2)(n^2+5)} < \sum_{n=1}^{\infty} \frac{5n^3}{n^5} = \sum_{n=1}^{\infty} \frac{5}{n^2}$$

$n^5+2n^4+5n^3+2n^2$

$(n+2)(n^2+5) = n^3+2n^2+5n+2$

p-series

$p=2$

(converge)

Feb 22-1:15 PM

9.5 b absolute vs. conditional convergence
intervals of convergence

absolute convergence: if $\sum |a_n|$ converges
then $\sum a_n$ converges

or

conditional convergence: $\sum |a_n|$ diverges
definition
and $\sum a_n$ converges
probably use AST

or

both diverge

Feb 22-1:23 PM

does $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converge absolutely, conditionally or neither?

1. absolute convergence $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} \ldots$

harmonic, diverges

2. conditional? yes $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots$

alt. harmonic

converges by AST

AST
✓1. signs alternate
✓2. |terms| get smaller
✓3. $a_n \to 0$

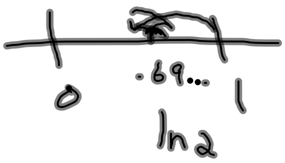
Feb 22-1:30 PM

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

let $x=1$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$$

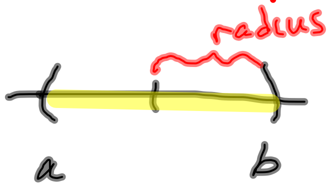
alt harmonic



Feb 22-1:41 PM

interval of convergence · values of x for which the series converges

ioc



1. ~~consider~~ find the middle — ratio test
2. evaluate the endpoints — AST

Feb 22-1:47 PM

Ex 6a find the interval of convergence for $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{2n}$

1. ratio test: $\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{2(n+1)} \cdot \frac{2n}{x^{2n}} \right|$

$\lim_{n\to\infty} \left| \frac{x^{\cancel{2n+2}}}{\cancel{x^{2n}}} \cdot \frac{2n}{2n+2} \right| = x^2 < 1$

$\frac{2n}{2n+2} \to 1$

$\sqrt{x^2} < \sqrt{1}$

$|x| < 1$

$\boxed{-1 \le x \le 1}$

$\frac{x^{2n+2}}{x^{2n}} = x^{2n+2-2n} = x^2$

2. endpts let $x = 1$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

3 let $x = -1$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{2n}}{2n} = \frac{1}{2} - \frac{1}{4} + \frac{1}{6} \dots$

converges by AST

Feb 22-1:54 PM